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A Data Driven Solution to the Dark Matter Problem

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Abstract

A data driven solution to the dark matter problem is presented. This short and self-contained overview is intended for a wide audience, with full technical details available in the cited references. We present redundant, independent and consistent measurements of the dark matter particle comoving root-mean-square velocity $v_{hrms}(1)$, or equivalently, of the dark matter temperature-to-mass ratio. These measurements agree with the "no freeze-in and no freeze-out" scenario of spin zero dark matter that decouples early on from the Standard Model sector, e.g. spin zero dark matter coupled to the Higgs boson or to the top quark.[1]

Keywords: Warm Dark Matter, Galaxy Rotation Curves, Galaxy UV Luminosity, Dwarf Galaxies

About this paper

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INTRODUCTION

Most of the matter in the universe, 84.3 ± 0.2 % [1], is in a "dark matter" form that has been "observed" only through its gravitational interaction. As far as we know, this dark matter does not have any other interaction, at least down to the current sensitivity of our experiments and observations. Fritz Zwicky in 1933 found that the matter in the Coma cluster of galaxies greatly exceeds the mass in stars [2]. According to the book by Stefano Profumo [3], the mass m_h of dark matter particles is unknown over 90

1 Presented at the 4th World Summit on Exploring the Dark Side of the Universe, La Reunion, November 7-11 2022

orders of magnitude! In summary, we know exactly how much dark matter there is, but do not have the foggiest idea what it is.

This article presents an overview of a data driven solution to the dark matter problem.

MEASUREMENTS

Let us assume that dark matter is a gas of particles that is ultra-relativistic in the early universe. As the universe expands, dark matter cools, and becomes non-relativistic. We expect that this non-relativistic gas is classical, i.e. has negative chemical potential [4] [5]. Let $v_{h\mathrm{rms}}(a)$ be the root-mean-square velocity of the non-relativistic dark matter particles. $a(t)$ is the expansion parameter of the universe at time t, normalized to $a(t_0)$= 1 at the present time.[2] $v^2_{h\mathrm{rms}}(a)$ is proportional to the dark matter temperature-to-mass ratio. As the universe expands, dark matter cools, $v_{h\mathrm{rms}}(a)$ varies in proportion to 1/a [6], and the dark matter density $\rho_h(a)$ varies in proportion to 1/a^3, so

$$v_{h\mathrm{rms}}(1) \equiv v_{h\mathrm{rms}}(a)a = v_{h\mathrm{rms}}(a)\left[\frac{\rho_h(1)}{\rho_h(a)}\right]^{1/3} \tag{1}$$

does not depend on a. In other words, we say that $v_{h\mathrm{rms}}(1)$ is an "adiabatic invariant".

It turns out that, to unravel the dark matter mystery, we need to measure the adiabatic invariant $v_{h\mathrm{rms}}(1)$, and the related observable $k_{\mathrm{fs}} \equiv 2\pi/\lambda_{\mathrm{fs}}$, that we now explain. Due to the velocity dispersion $v_{h\mathrm{rms}}(a)$, dark matter particles free-stream in and out of density minimums and maximums, erasing primordial density fluctuations of "comoving" wavelength less than approximately λ_{fs}[4] [7]. (Since wavelengths grow in proportion to $a(t)$, it is customary to refer the wavelength to the present time, i.e. $a(t_0)$= 1, hence the word "comoving".) k_{fs} is the comoving cut-off wavenumber due to dark matter free-streaming. The theoretical relation between the two observables, $v_{h\mathrm{rms}}(1)$ and k_{fs}, is summarized in Table 1 [4] [7].

To measure k_{fs} we compare observed and predicted galaxy rest-frame ultra-violet luminosity distributions, and observed and predicted galaxy stellar mass distributions. An example, corresponding to "redshift" $z = 6$, or equivalently, expansion parameter a = 1/(1+z) = 1/7, is shown in Figure 1. Note that warm dark matter free-streaming attenuates small scale fluctuations and therefore reduces the numbers of galaxies with low mass. From this, and similar figures at redshifts z = 4, 8 and 10, we obtain $k_{\mathrm{fs}} = 2.0^{+0.8}_{-0.5}$ Mpc^{-1} [17].

2 To understand the expansion parameter $a(t)$, imagine a baloon covered with dots. As the baloon is inflated, the distances between neighboring dots, i.e. galaxies, increase in proportion to $a(t)$.

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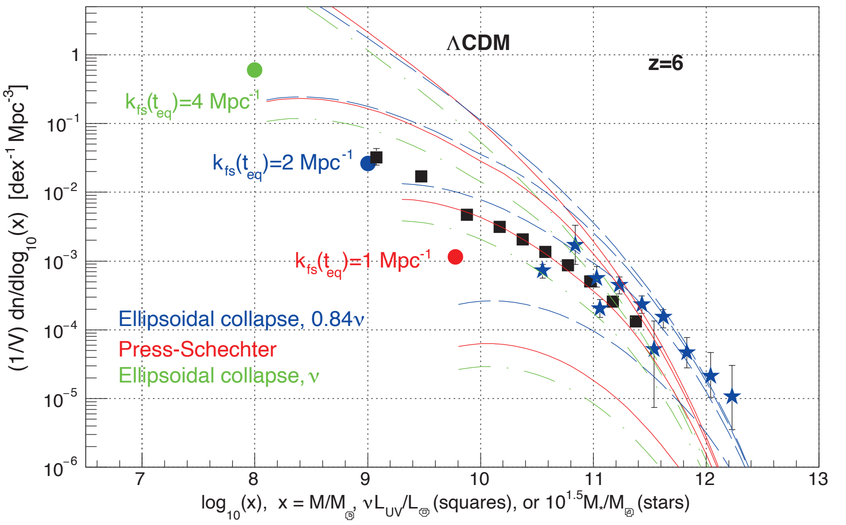

Figure 1. Shown are distributions of x, where x is the observed galaxy stellar mass $M_*/M_\odot$ times $10^{1.5}$ (stars) [8] [9] [10], or the observed galaxy UV luminosity $\nu L_{\rm UV}/L_\odot$ (squares) [11] [12] [13] (corrected for dust extinction [14] [15]), or the predicted linear total (dark matter plus baryon) mass $M/M_\odot$ (lines), at redshift $z=6$. The symbol $\odot$ means "sun". The Press-Schechter prediction, and its Sheth-Tormen ellipsoidal collapse extensions, correspond, from top to bottom, to the warm dark matter free-streaming cut-off wavenumbers $k_{\rm fs}=1000$, 4, 2 and 1 Mpc^{-1}. The round red, blue and green dots indicate the velocity dispersion cut-offs of the predictions [16] at $k_{\rm fs}=1$, 2 and 4 Mpc^{-1}, respectively. Presenting three predictions illustrates the uncertainty of the predictions. Note that the data agree with predictions for $k_{\rm fs}\approx 2$ Mpc^{-1}.

Table 1. Calculated relation between the adiabatic invariant $v_{hrms}(1)$, and the comoving cut-off wavenumber $k_{\rm fs}$, due to dark matter free-streaming [4] [7]. "Mega-parsec" (Mpc) is a unit of length used in cosmology.

$v_{hrms}(1)$	k_{fs}
750 m/s	1 Mpc^{-1}
490 m/s	1.53 Mpc^{-1}
370 m/s	2 Mpc^{-1}
190 m/s	4 Mpc^{-1}
0.75 m/s	1000 Mpc^{-1}

To measure the adiabatic invariant $v_{hrms}(1)$ we note the following. Consider a free observer in a density peak in the early universe. This observer "sees" dark matter expand adiabatically, i.e. conserving $v_{hrms}(1)$, due to the expansion of the universe, reach maximum expansion, followed by adiabatic compression into the core of the galaxy due to gravitational attraction. The core of the galaxy forms adiabatically if dark matter is warm as we have assumed, i.e if $v_{hrms}(1)$ is greater than zero [18]. Rotation and relaxation, due to galaxy collisions and mergers, increase the observed $v'_{hrms}(1)$ above the true $v_{hrms}(1)$ [6]. So, as long as rotation and relaxation remain negligible, we predict that the adiabatic

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invariant in the core of the galaxy is the same as in the early universe, and so should be the same for all galaxies (with negligible rotation and relaxation). The adiabatic invariant in the core of a spiral galaxy can be obtained from the observed rotation curves of neutral atomic hydrogen gas or of stars, together with infrared and visible images [19]. The distribution of the adiabatic invariant measured in several dwarf galaxies is shown in Figure 2. We obtain a narrow peak with $v_{hrms}(1) = 406 \pm 69$ m/s [6]. The few galaxies with $v'_{hrms}(1)$ to the right of this peak have significant rotation and/or relaxation.

We note, from Table 1, that the measurements of k_{fs} and $v_{hrms}(1)$ are consistent with each other. We conclude that, 1) $v_{hrms}(1)$ in the core of galaxies (corrected for dark matter rotation and relaxation) is of cosmological origin, as inferred form the narrowness of the peak in Figure 2, and as predicted for warm dark matter [18] [21]; and 2) that k_{fs} is indeed due to warm dark matter particle free-streaming. Additional observables studied, that obtain consistent results, are the redshift of first galaxies [16], the ultra-violet luminosity of first galaxies [17], and the related "reionization optical depth" [17]. For a summary of measurements see [17].

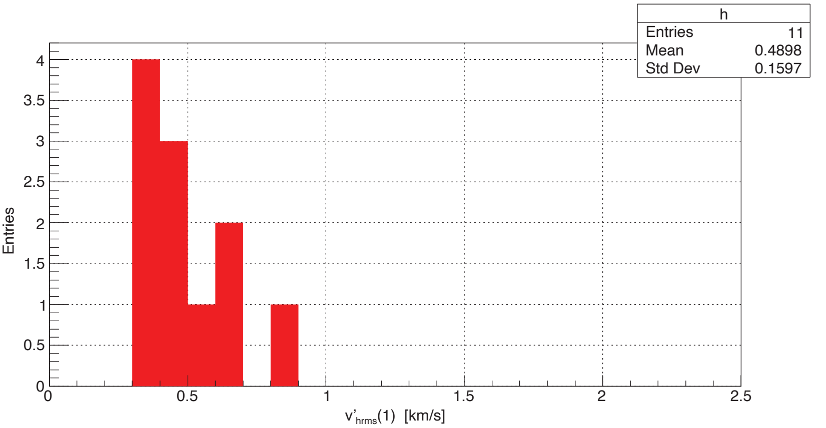

Figure 2. Distribution of $v'_{hrms}(1)$, i.e. the adiabatic invariant before the dark matter rotation and relaxation correction, of 11 dwarf galaxies. These corrections can only be negative, and so are negligible in the peak at $v'_{hrms}(1) \approx v_{hrms}(1) \approx 0.4$ km/s. Data from [20].

These measurements obtain the dark matter temperature-to-mass ratio, not separately the temperature or mass. At this point we have already learned something: dark matter models that predict $v_{hrms}(1)$ in disagreement with these measurements can be ruled out (if the measurements are correct). In particular, cold dark matter candidates, such as "WIMPS" and "axions", are disfavored. Let us mention that the entire analysis is data driven, with full details, and the sources of all data and measurements, available in the references.

COLD AND WARM DARK MATTER

The cold dark matter ΛCDM theory, with only six parameters, is in spectacular agreement with the large scale phenomena of the "Cosmic Microwave Background Radiation", "Baryon

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Acoustic Oscillations", and the large scale matter distribution of the universe. However, there are, or appear to be, tensions with small scale phenomena, less than the size of the Galaxy, known as the "Core-Cusp Problem", the "Missing Satellites Problem", and the rest frame ultra-violet luminosity cut-off required to not exceed the "Reionization Optical Depth" measured by the Planck collaboration. Adding to the cold dark matter $\Lambda\mathrm{CDM}$ theory one more parameter, namely the adiabatic invariant $v_{h\mathrm{rms}}(1)$, obtains the warm dark matter $\Lambda\mathrm{WDM}$ cosmology. It turns out that $\Lambda\mathrm{WDM}$ agrees with $\Lambda\mathrm{CDM}$ on large scales, and, with the measured $v_{h\mathrm{rms}}(1)$ and k_{fs} indicated above, solves all of the mentioned small scale tensions [18] [22].

NO FREEZE-IN AND NO FREEZE-OUT

And now comes the miracle. The measured k_{fs} and $v_{h\mathrm{rms}}(1)$, or equivalently, the measured dark matter temperature-to-mass ratio, happen to coincide with the predictions of the "no freeze-in and no freeze-out" scenario of spin zero dark matter particles that decouple early on from the Standard Model sector, e.g. dark matter particles coupled to the Higgs boson, or to the top quark. Let me explain. "No freeze-in" means that dark matter reaches thermal and diffusive, i.e. chemical, equilibrium, in the early universe, with the particles of the "Standard Model of Quarks and Leptons", or, loosely speaking, with ordinary matter. Dark matter then decouples from the Standard Model sector while still ultra-relativistic. "No freeze-out" means that, when dark matter becomes non-relativistic, its mutual interactions are so weak that these particles do not annihilate each other. An example of spin zero dark matter coupled to the Higgs boson is shown in Figure 3.

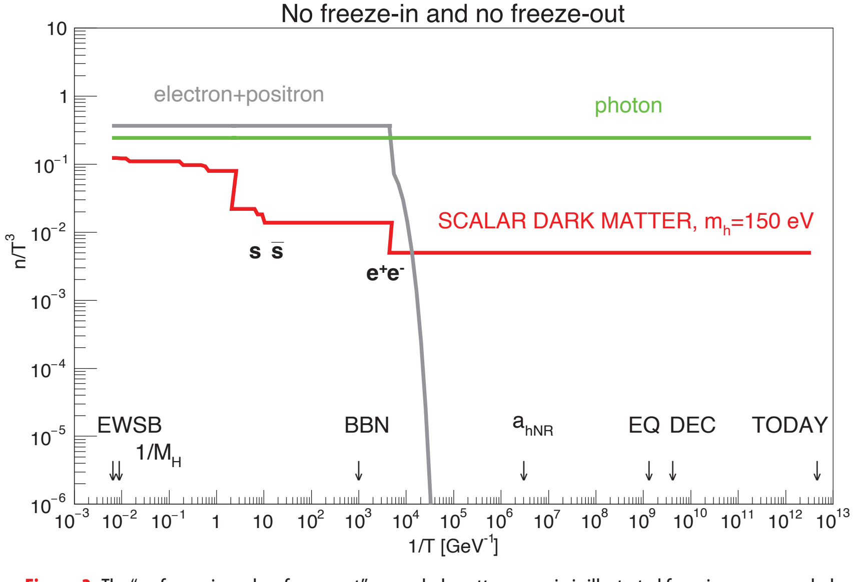

Figure 3. The "no freeze-in and no freeze-out" warm dark matter scenario is illustrated for spin zero warm dark matter particles coupled to the Higgs boson. T is the photon temperature, and the n's are particle number densities, i.e. numbers of particles per unit volume. The abbreviations stand for "Electro-Weak Symmetry Breaking", "Big Bang Nucleosynthesis", "EQuivalence" of matter and radiation densities, and "DECoupling" of photons from Standard Model matter. Dark matter particles become non-relativistic at $a_{h\mathrm{NR}}$. Time advances towards the right.

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If dark matter is created by Higgs boson annihilation or decay, then freeze-in and/or freeze-out lowers the dark matter temperature-to-mass ratio. In this case, the *measured* $v_{hrms}(1)$ is *evidence* of no freeze-in and no freeze-out, not a coincidence.

If spin zero dark matter is indeed coupled to the Higgs boson [23], then the no freeze-in and no freeze-out scenario, together with the measured dark matter density and measured cosmic microwave background temperature, predicts $v_{hrms}(1) = 490 \pm 10$ m/s, $k_{fs} = 1.53 \pm 0.03$ Mpc^{-1}, and the mass of the dark matter particles $m_h = 150 \pm 2$ eV [4], or about 1/3400 of the electron mass. See the Appendix for details of the calculations.[3] For couplings to other massive Standard Model particles, the predictions vary slightly. The data disfavors spin one-half and spin one dark matter coupling to Standard Model particles, since their predicted $v_{hrms}(1)$ is greater than observations [17].

The minimal extension of the Standard Model to obtain the no freeze-in and no freeze-out scenario of dark matter is to add a real scalar field S with Z_2 symmetry $S \leftrightarrow -S$ that couples to the Higgs boson ϕ as $\mathcal{L}_{S\phi} = -\frac{1}{2}\lambda_{hS}\left(\phi^\dagger\phi\right)S^2$ [23]. The fields ϕ [25] and/or S can be the "inflaton" that drives inflation. For example, for a reheating temperature $T \approx 10^{15}$ GeV, equilibrium between ϕ and S is obtained with $\lambda_{hS} \gtrsim 10^{-7}$.

CONCLUSIONS

In summary, we redundantly measure the dark matter temperature-to-mass ratio, and arrive at a plausible, data driven, and detailed solution to the dark matter problem, i.e. the no freeze-in and no freeze-out scenario of scalar dark matter that decouples early on from the Standard Model sector.

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3 Some publications rule out such small m_h, apparently because they neglect non-linear regeneration of small scale structure [4] [22] [24]. Let us mention that according to "The Review of Particle Physics" [1], limits on dark matter particle mass are $m_h > 70$ eV for fermions, and $m_h > 10^{-22}$ eV for bosons, and not several keV.

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APPENDIX A: THE NO FREEZE-IN AND NO FREEZE-OUT SCENARIO

We present a summary of Section 3 of [4] to make the present article self-contained.

We assume that the ultra-relativistic dark matter is in thermal and diffusive equilibrium with the Standard Model sector in the early universe. In particular we assume that the ultra-relativistic dark matter has zero chemical potential. We assume that dark matter decouples from the Standard Model sector while still ultra-relativistic. As the universe expands, dark matter cools and becomes non-relativistic. We assume that the ultra-relativistic Bose-Einstein or Fermi-Dirac momentum distribution of dark matter relaxes to the corresponding non-relativistic distribution, as justified in [5], thereby acquiring a negative chemical potential [5].

Let T_h/T be the dark matter-to-photon temperature ratio after decoupling of neutrinos, and after e^+e^- annihilation, and before dark matter becomes non-relativistic. This ratio is [5] [4]

$$\frac{T_h}{T} = \left(\frac{43}{11 g_{\text{dec}}} \right)^{1/3}, \quad (2)$$

where $g_{\text{dec}} = \sum N_b + (7/8) \sum N_f$ at decoupling of dark matter from the Standard Model sector [1]. N_f (N_b) is the number of fermion (boson) spin polarizations. The ratio of number densities of dark matter particles and photons, after e^+e^- annihilation until the present time, is [5] [4]

$$\frac{n_h}{n_\gamma} = \frac{43 g'_h}{22 g_{\text{dec}}}, \quad (3)$$

where $g'_h = N_b + 3N_f/4$ for the dark matter [5]. Then, at the present time,

$$\Omega_c h^2 = \frac{n_h m_h h^2}{\rho_{\text{crit}}} \approx \frac{114}{g_{\text{dec}}} \frac{g'_h}{1.5} \frac{m_h}{\text{keV}} \quad (4)$$

determines the dark matter particle mass m_h corresponding to no freeze-in and no freeze-out.

For fermions, from equation (26) of [5], we obtain

$$m_h = 108 \left(\frac{0.76 \text{ km/s}}{v_{hrms}(1)} \right)^{3/4} \left(\frac{2}{N_f} \right)^{1/4} \text{ eV}, \tag{5}$$

$$\frac{T_h}{T} = 0.336 \left(\frac{v_{hrms}(1)}{0.76 \text{ km/s}} \right)^{1/4} \left(\frac{2}{N_f} \right)^{1/4}. \tag{6}$$

For bosons, from equation (28) of [5], we obtain

$$m_h = 108 \left(\frac{0.76 \text{ km/s}}{v_{hrms}(1)} \right)^{3/4} \left(\frac{1}{N_b} \right)^{1/4} \text{ eV}, \tag{7}$$

$$\frac{T_h}{T} = 0.385 \left(\frac{v_{hrms}(1)}{0.76 \text{ km/s}} \right)^{1/4} \left(\frac{1}{N_b} \right)^{1/4}. \tag{8}$$

Note that the measurement of the adiabatic invariant $v_{hrms}(1)$ allows a determination of the dark matter particle mass m_h, and of the ratio T_h/T (that determines the dark matter decoupling temperature, if sufficiently precise).

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