

## NLOS Mitigation for Mobile Subscriber Positioning Systems By Weighting measures and Geometrical Restrictions

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Editado por/Edited by: Cesar Zambrano, Ph.D.

Recibido/Received: 2015/04/09. Aceptado/Accepted: 2015/05/10.

Publicado en línea/Published on Web: 2015/12/30. Impreso/Printed: 2015/12/30.

### Abstract

This paper studies the Non-Line-Of-Sight condition mitigation issue in mobile subscriber positioning systems by weighting Time-Of-Arrival measures and applying geometrical restrictions. Particularly, this work departs from a more exact characterization of the signal statistics to achieve weighting factors able to reach a more effective mitigation, and consequently a more accurate mobile subscriber positioning. In addition, restrictions based on the cell geometry are incorporated as a complementary refinement method. Therefore, three new methods with better properties than those taken from the literature and used as reference are introduced. These approaches are evaluated within a realistic simulation scenario.

**Keywords.** NLOS mitigation, TOA based positioning systems, WLLS, geometrical restrictions, wireless sensor networks.

### Mitigación de la Condición NLOS para Sistemas de Posicionamiento de Suscriptor Móvil usando Ponderación de Medidas y Restricciones Geométricas

### Resumen

Este artículo estudia el problema de la mitigación de la condición de ausencia de Línea de Visión en sistemas de posicionamiento de suscriptor móvil utilizando la ponderación de medidas de Tiempo de Arribo y la aplicación de restricciones geométricas. En particular, este trabajo parte de una caracterización más exacta de la estadística de la señal, para conseguir factores de ponderación con capacidad de alcanzar una mitigación más efectiva, y consecuentemente un posicionamiento más preciso del suscriptor móvil. Adicionalmente, se incorporan restricciones basadas en la geometría de la celda como método de refinamiento complementario. Se presentan en consecuencia tres nuevos métodos de posicionamiento con mejores propiedades que los tomados de la literatura y usados como referencia. Estas técnicas se evalúan dentro de un escenario de simulación realista.

**Palabras Clave.** Mitigación NLOS, sistemas de posicionamiento basados en Tiempo de Arribo (TOA), WLLS, restricciones geométricas, redes de sensores inalámbricos.

### Introduction

Positioning of a mobile subscriber is a complex task with the capability of adding value to services and applications. The knowledge of the positioning of a certain device is important, but the applications and services to be provisioned from the awareness of that position will probably be more useful from the perspective of the user, and consequently more impacting to our society. Therefore a close relationship and dynamism are associated to: a) wireless communications that provide user ubiquity, b) positioning technologies that refer to the ways in which the measured signals from network

and/or a mobile subscriber are treated to compute its position, and c) Location Based Services (LBS). In fact, LBS are a key piece of this dynamism, not just because LBS are hunger of more resources from network devices but also because they take advantage of the virtues of new communication technologies to construct new possibilities of relation among users, between users and service providers, and also between providers and third parties such as contents' providers. Neither it is strange that all these systems' elements are object of permanent research and keep in permanent revision [1–14].

This paper focuses in network based positioning technologies and particularly in the mitigation of an important issue, which strongly degrades the accuracy of the subscriber due to the specific propagation conditions of the wireless signals, known as Non Line Of Sight (NLOS) condition. Furthermore, enabling positioning technologies based on Time Of Arrival (TOA) measures will be used along this document to illustrate the problem and the means employed to mitigate it, within a simulation environment developed to reproduce realistic wireless propagation conditions.

### The Positioning Problem

Torrieri [15] established the statistical principles for passive location systems, and generalized this problem assuming the measurements vector  $\mathbf{m}$ , may be view as a function of the position vector  $\mathbf{x}$ , plus additive noise  $\mathbf{n}$ , as in (1):

$$\mathbf{m} = f(\mathbf{x}) + \mathbf{n} \quad (1)$$

The actual nature of  $f(\mathbf{x})$  depends of the type of the measurements' set used for computing the positioning, and in the case of range-based methods such as TOA, TDOA and RSSI, it is a nonlinear function related to the range among the subscriber position and those BS's participating in the positioning. The expression for TOA is exhibited in (2); where  $L$  refers to the number of BSs,  $\mathbf{x}=(x,y)$  to the true coordinates of the subscriber position, and  $\mathbf{r}_i=(x_i,y_i)$  denotes to the position of BS $_i$  used as reference.

$$f_{TOA_i}(\mathbf{x}) = \|\mathbf{x} - \mathbf{r}_i\|; \quad \forall i = 1, 2, \dots, L \quad (2)$$

The general nonlinear solution corresponds to the minimization of the noise in (1) by using Maximum Likelihood (ML), Nonlinear Least Squares (NLS) or the Weighted Nonlinear Squares (WNLS) approach [18]. The WNLS solution requires minimization of a cost function, where the actual function  $J_{WNLS}$  depends of the type of measurements employed and its general form is shown in (3).

$$J_{WNLS}(\mathbf{x}) = [\mathbf{m} - f(\mathbf{x})]^T \mathbf{C}_n^{-1} [\mathbf{m} - f(\mathbf{x})] \quad (3)$$

with  $\mathbf{C}_n = E\{\mathbf{nn}^T\}$

Performing a ML solution for this estimation problem requires noise statistics, and when the measurement noise  $\mathbf{n}$  in (1) is zero-mean and Gaussian distributed with covariance matrix  $\mathbf{C}_n$ , it may be easily shown as ML scheme reduces to the WNLS solution, and finally to NLS when measurement noise is statistically independent and identically distributed. The ML approach requires a high complexity when grid search is achieved, and therefore global solution may not be guaranteed, but in general

its accuracy is the highest, especially when  $\mathbf{C}_n$  is also a function of subscriber position because if it is the case, the cost function includes the term  $\ln[\det(\mathbf{C}_n)]$  that avoids the selection of positions with large uncertainty [16]. On the other hand, NLS does not require noise statistics but also involves the same issues as ML.

The vector function  $f(\mathbf{x})$  in (1), which relates the position in the real world  $\mathbf{x}$ , to the measurement space  $\mathbf{m}$  is nonlinear in general, but it becomes linear through a Taylor series expansion around an arbitrary point  $\mathbf{x}_0$  located near the subscriber position, and it requires the Jacobian matrix for  $f(\mathbf{x})$  evaluated in  $\mathbf{x}_0$ , for the  $L$  measurements [15–17].

Assuming a zero mean Gaussian distribution for the noise vector  $\mathbf{n}$ , a Linearized Least Squares position Estimator was proposed by Torrieri [15]. This solution uses the iterative algorithm known as Gauss-Newton method for reaching the cost function minimum, but others methods such as Newton-Raphson or Steepest Descent may be used instead [16, 17].

It is also possible to convert the nonlinear formulation in (1) into a set of linear equations with the form in (4) under the assumption that measurements errors are small enough. The actual form of matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  depends of the kind of measurements chosen by the enabling technology. Expressions (5) and (6) exhibit the corresponding structures for the case of TOA based positioning systems when the controlling BS is assumed to be at the origin, and where  $c$  refers to the light speed. Linear procedures include to Linear Least Squares (LLS), Weighted LLS (WLLS) and the subspace estimators [18]. LLS and WLLS are the linear counterparts of NLS and WNLS respectively.

$$\mathbf{Ax} = \mathbf{b} \quad (4)$$

$$\mathbf{A}_{TOA} = \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_L & y_L \end{bmatrix} \quad (5)$$

$$\mathbf{b}_{TOA} = \frac{1}{2} \begin{bmatrix} r_2^2 - m_2^2 + m_1^2 \\ r_3^2 - m_3^2 + m_1^2 \\ \vdots \\ r_L^2 - m_L^2 + m_1^2 \end{bmatrix} \quad (6)$$

with  $r_i^2 = x_i^2 + y_i^2; \quad \forall i = 1, \dots, L$   
and  $m_i = c \cdot t_i; \quad \forall i = 1, 2, \dots, L$

The LLS solution is shown in (7), and this procedure may also be applied to TDOA with certain modifications [17].

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \quad (7)$$

In addition, it is worth to note as all of these LLS procedures dismiss the noise statistics and therefore will achieve their most performance in case of low noise patterns.

The WLLS approach emerges from including a weighting matrix  $\mathbf{W}$ , within the cost function as it is shown in (8). This assessment matrix is precisely the inverse of the covariance noise matrix, and after cost function minimization, the WLLS estimator is achieved as in (9):

$$J_{WLLS} = E \left\{ (\mathbf{Ax} - \mathbf{b})^T \mathbf{W} (\mathbf{Ax} - \mathbf{b}) \right\} \quad (8)$$

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{WA})^{-1} \mathbf{A}^T \mathbf{Wb} \quad (9)$$

The particular weighting matrix  $\mathbf{W}$  depends of the type of measurements, and it is usually dependent of the distances between subscriber and BSs due to transformations performed during formulation of the linear system. Expression (10) exhibits this matrix for the case of TOA based positioning. In this case, the set of required distances  $d_i$  may be replaced for the set of measurements  $m_i$ , but in case of TDOA or AOA, a LLS procedure should be firstly performed to estimate subscriber position to properly figure out the distances required within  $\mathbf{W}$ , and a second step is also required to achieve the refined WLLS position estimation. In case of TDOA, the weighting matrix in (11) may also be also used as first step when a WLLS initialization is preferred. Furthermore, an iterative process may be performed in order to minimize the function cost in (8) and achieve the maximum accuracy, and consequently the Best Linear Unbiased Estimator (BLUE) algorithm, but in general a two-step LS algorithm is adequate. Alternative formulations for the positioning problem are possible, but results in terms of accuracy are not important [22].

$$\begin{aligned} \mathbf{W}_{TOA} &= [E \{ \mathbf{ee}^T \}]^{-1} \\ &= \frac{1}{4} \text{diag} \left( \frac{1}{\sigma_{TOA,2}^2 d_2^2}, \dots, \frac{1}{\sigma_{TOA,L}^2 d_L^2} \right) \quad (10) \\ \text{with } d_i^2 &= \|\mathbf{x} - \mathbf{r}_i\|^2; \quad \forall i = 2, 3, \dots, L \end{aligned}$$

$$\mathbf{W}_{TDOA}^{-1} = C_{TDOA} = \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 2 \end{bmatrix} \quad (11)$$

Whichever would be the positioning technique employed, it should be kept in mind that every set of measurements performed by a sensor reduces the positioning to a region shaped in a way related to the nature of the measurements, a feasible region. A TOA based positioning corresponds to a circular-circular system, whilst a hyperbolic-hyperbolic system is characteristic of a TDOA

based positioning. Furthermore, it must be noted as hybrid techniques exhibit a better behavior than homogeneous ones since it is a well-known principle that errors achieved from a particular positioning technique may be overcome with the application of another one [23]. Positioning accuracy may also take advantage of spatial diversity and mobile system's dynamic [19, 20, 30]. In fact, Kalman Filter [21, 29] and its variants [25, 28] have been probed their efficacy by using the mobility dynamics.

### The signal model and the NLOS issue in the mobile subscriber positioning problem

Due to the presence of obstacles between emitter and transmitter, received signal is scattered in space and time, and the LOS component may be strongly degraded or even completely shadowed. However, receiver generally uses the most powerful arriving components and therefore, in case of shadowing, LOS component is eventually discarded, and measures are achieved under a NLOS condition. This NLOS multipath signal travels a longer distance than the LOS component to reach the receiver and consequently the measures are biased as it is shown in (12), where  $\mathbf{q}$  is precisely the vector which contains biases due to NLOS.

$$\begin{aligned} \mathbf{m} &= f(\mathbf{x}) + \mathbf{n} + \mathbf{q} \\ \text{with } \mathbf{q} &= [q_1, q_2, \dots, q_L]^T \quad (12) \end{aligned}$$

These biases are positive random variables. When  $q_i=0$ , it refers to a LOS condition, and when  $q_i \gg |n_i|$ , it refers to a strong NLOS condition, being the latter the case commented along this paper. Bias nature is associated to propagation conditions, and in case of TOA based systems, it may be related directly to the Excess Delay through the Power Delay Profile (PDP). Furthermore, the Greenstein model [9] has been considered to perform this characterization, since it adjusts to several measurement - based models and incorporates their information into a small number of parameters to characterize the path-gain/delay spread propagation channel, and even this model has been incorporated to COST-231 and eventually to the COST-259 Directional Channel Model [12, 25].

NLOS environments are modeled using an exponential distribution for the excess delay for a particular location as it is shown in (13), and the Greenstein model characterizes the required RMS Delay Spread  $\tau_{rms}$  in (14) as a random variable and also as a function of the distance between emitter and receiver, where  $\xi$  is a lognormal random variable. Hence,  $\Xi = 10 \log(\xi)$ , is a zero mean Gaussian variable over the terrain, with a standard deviation  $\sigma_\xi$  that lies between 2 and 6 [dB]. Furthermore,  $T_1$  corresponds to the median value of  $\tau_{rms}$  at  $d=1$  [km], and  $\varepsilon$  is an exponent that lies between 0.5-1.0. It has

been set to 0.5 for the simulations exhibited in this document.

$$f_{\tau}(\tau) = \frac{1}{\tau_{rms}} \exp\left[-\frac{\tau}{\tau_{rms}}\right] u(\tau) \quad (13)$$

$$\tau_{rms} = T_1 d^{\epsilon} \xi \quad (14)$$

The Greenstein model also includes the gain path  $g$ . This gain is computed with the use of the expression in (15), where  $d$  is the distance in kilometers,  $G_1$  is the median value of  $g$  at  $d=1$  [km],  $\beta$  is the loss path propagation factor which lies between 3 and 4, and  $\mathbf{x}$  is a lognormal random variable. Therefore,  $\mathbf{X}=10\log(\mathbf{x})$  is a zero mean Gaussian with a standard deviation  $\sigma_x$  between 6 and 12 [dB]. And finally, the correlation factor among  $\mathbf{X}$  and  $\Xi$  has been set as  $\rho=-0.7$  [9]. Therefore  $E\{\mathbf{X},\Xi\} = \rho \cdot \sigma_x \cdot \sigma_{\xi}$ .

$$\mathbf{g} = \frac{G_1}{d^{\beta}} \mathbf{x} \quad (15)$$

The mean and the standard deviation for the RMS Delay Spread modeled as in (14) are presented as a function of distance within (16) and (17) respectively, being  $m_z$  and  $\sigma_z$  the mean and standard deviation of the scaled random variable  $\mathbf{Z}=\Xi \cdot \ln(10)/10$ . These expressions are derived in the Annex.

$$E\{\tau_{rms}\} = T_1 d^{\epsilon} e^{m_z + \sigma_z^2/2} \quad (16)$$

$$\sigma_{\tau_{rms}} = \sqrt{\text{var}\{\tau_{rms}\}} = k T_1 d^{\epsilon} \quad (17)$$

Since the standard deviation for Delay Spread in (17) increases proportionally to  $T_1 d^{\epsilon}$ , it is reasonable to modify the weights in the WLLS algorithm in (10) as in (18) to include this information as a mean for NLOS mitigation. This new version of the WLLS algorithm will be called NLOS WLLS in this document.

$$\mathbf{W}_{NLOS} = \text{diag}\left(\frac{1}{T_1^2 d_2^{2(1+\epsilon)}}, \dots, \frac{1}{T_1^2 d_L^{2(1+\epsilon)}}\right) \quad (18)$$

In addition, and due the set of TOA measures are biased, a geometric mitigation will also be tested. This Geometric based mitigation will simply estimate subscriber position as the centroid of the resulting triangle from the intersection of the Circular Lines Of Position described by the measures achieved from the three nearest BS's.

Finally, two new algorithms resulting from the incorporation of Geometric restrictions to our proposed NLOS WLLS are also evaluated in the next section.

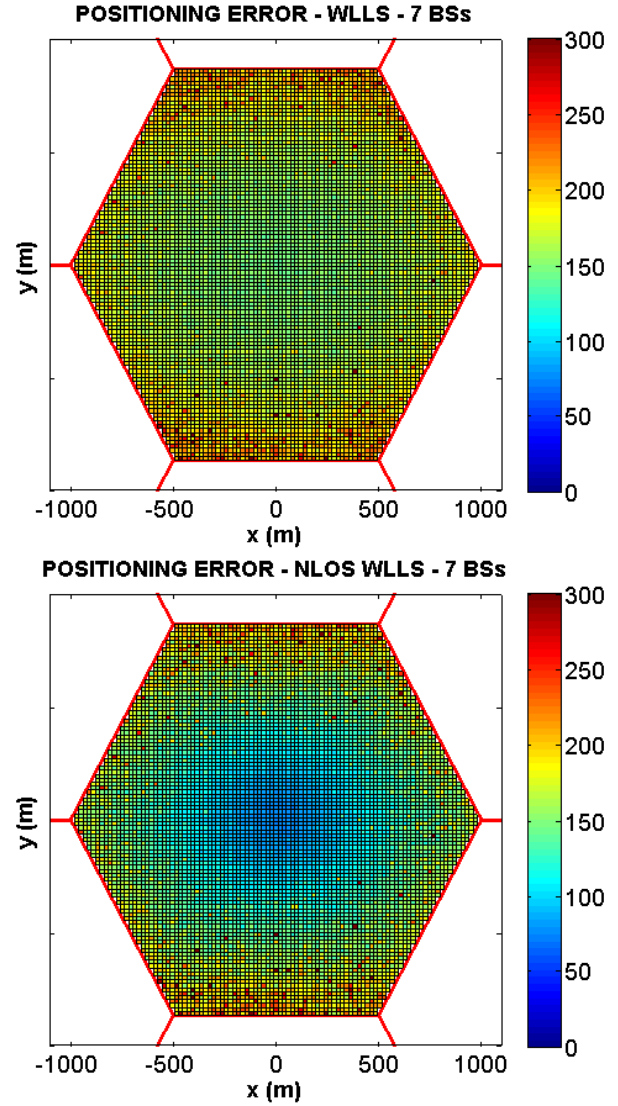


Figure 1: Subscriber Average Positioning Error for a dispersive NLOS environment,  $T_1=0.4[\mu s]$  and 7 BS's. A) Top: original WLLS algorithm. B) Bottom: NLOS WLLS method with  $\epsilon=1$ .

### Algorithms' Performance Evaluation

This section includes some simulations provided to evaluate several positioning algorithms and their capabilities to mitigate the NLOS condition.

To perform positioning evaluation, a simulation platform compounded for a seven hexagonal cell cluster has been considered. The control site is located at the coordinate system origin, and a rectangular grid has been constructed within the control cell to evaluate subscriber positioning algorithms' behavior for each point within the cell.

A realistic scenario is considered. It assumes that NLOS condition is present in the seven BS's. However, NLOS is assumed to be more moderated for the communications between the subscriber and the control site. For this latter BS, the Greenstein model is used as in the rest of sites with the only difference that the propagation losses factor  $\beta$  is reduced from 3.7 to 2.5. However

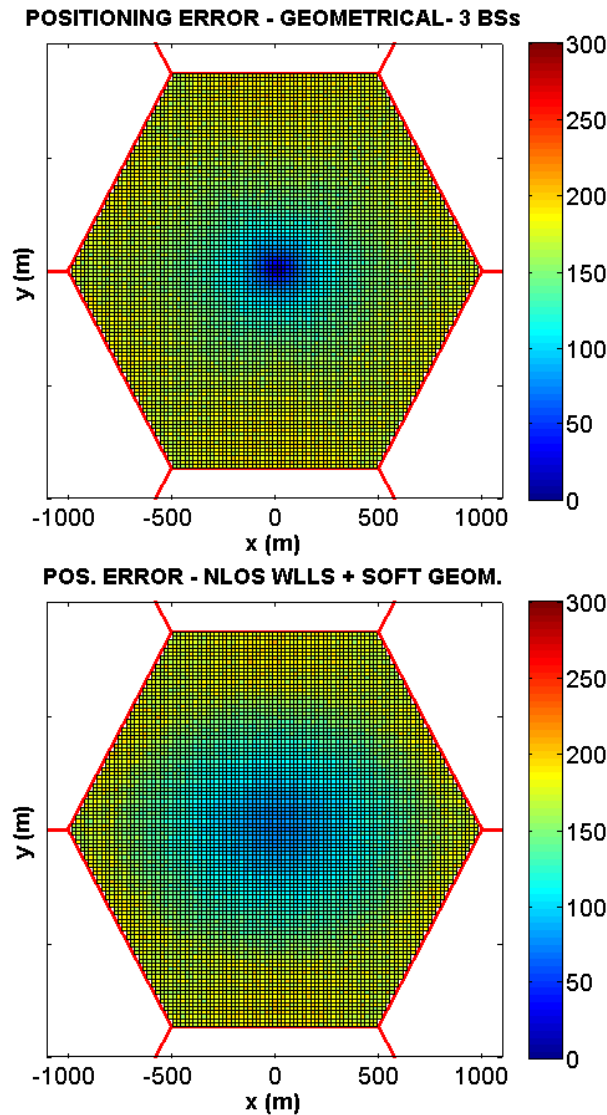


Figure 2: Subscriber Average Positioning Error for a dispersive NLOS environment. A) Top: Based on Geometrical Restrictions and 3 BS's. B) Bottom: 7 BS's - NLOS WLLS with  $\epsilon=1$  and Soft Geometrical Restrictions.

signal strength information is not relevant for results in this paper.

Particularly, the required parameters for the Greenstein model take the following values suitable for the urban case [9]:  $T_1=0.4$  [us],  $\epsilon=0.5$ ,  $\sigma_x=8.0$  [dB],  $\sigma_\xi=2.0$  [dB], and  $\rho=-0.75$ .  $T_1$  has been set in agree to the GTU COST 259 model [12, 26] and it may be considered a moderate dispersive environment.

Figure 1 and Figure 2 exhibit the average positioning errors for subscribers within a cell with radius  $R=1000$  [m] for several of the methods considered in this study. Particularly, Figure 1 shows the behavior from the application of both the original WLLS method described by (9) and (10), and the proposed NLOS WLLS algorithm with modified weights as in (18). Clearly the proposed method achieves a better NLOS mitigation especially near the control site, but it also exhibits accuracy degra-

ation in the boundaries.

Figure 2 on the other hand, shows the behavior of the considered Geometrical Positioning and the positive benefit of adding certain geometrical restrictions to the NLOS WLLS algorithm. In fact, the Geometrical Positioning exhibits a better behavior near the cell boundaries when it is compared with methods in Figure 1. Furthermore, when this information is added to the NLOS WLLS method, the new algorithm provides the best mitigation of them all.

To incorporate the information provided by the geometrical restriction, positioning is performed for the both basic algorithms; then, when NLOS WLLS procedure estimates that subscriber is located in the outer 30% portion of the cell, the positioning based on Geometrical restrictions is used to provide the subscriber position. If a hard fusion is performed, this latter position is used instead of the first. On the other hand, a soft decision implies to take the average of both estimations.

This second technique is the employed in simulation at the bottom of Figure 2.

Figure 3 compares the CDF's for the positioning error from various mitigation techniques commented along this paper. It includes the original WLLS algorithm, the proposed NLOS WLLS method, the positioning based on Geometrical restrictions, the Geometric Assisted Location Estimator (GALE) approach in [24], the Yi-Long algorithm in [27], and the two algorithms that introduce geometrical restrictions to the proposed NLOS WLLS method. All this algorithms use measures provided by the seven BS's with the exception of the algorithm based on Geometrical restrictions that use just the three nearest stations. These results confirms NLOS WLLS plus soft geometrical restrictions algorithm as the best among the whole set of implemented methods. In fact, this algorithm exhibits average positioning errors below 162[m] in 70% of the cases, and below 191 [m] in 95% of the cases. NLOS WLLS plus hard geometrical restrictions algorithms also makes a good job and registers average positioning errors below 172 [m] in 70% of the cases and below 188 [m] in the 95% of the cases. These latter values are slightly better than those provided by the Geometric algorithm which exhibits average positioning errors below 175 [m] in the 70% of the cases and below

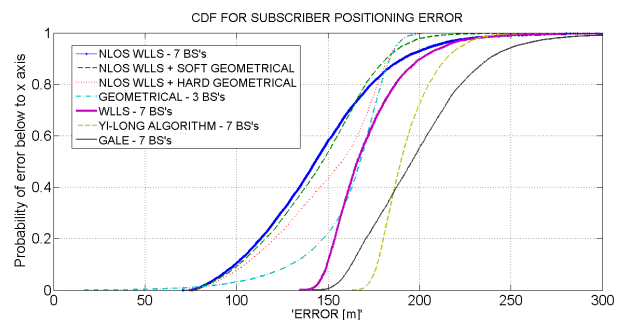


Figure 3: Cumulative Distribution Function for subscriber positioning error and different methods.



188 [m] in the 95% of the cases although a good portion of its data registers up to 25 [m] of additional error. In addition, the original NLOS WLLS algorithm achieves an average positioning error below 162 [m] in the 70% of the cases and below 208 [m] in the 95% of the cases, whilst the traditional WLLS algorithm reaches an average positioning error below 179 [m] for the 70% of the cases and below 215 [m] for the 95% of the cases. It means that NLOS WLLS plus soft Geometrical restrictions provides an additional mitigation of around of 15-45 [m] when it is compared with the traditional WLLS method in this moderate dispersive environment. Simulations performed within a more dispersive environment have shown as positioning degrades for all methods but also as these gains enhance. In addition, Yi-Long approach [27] and particularly GALE method [24] exhibited a poor performance within our NLOS simulation scenery where Gaussian measurement noise has not been included. In fact, our best approach provides an additional mitigation of around 35-70[m] respect to Yi-Long approach and around 50-70[m] respect to GALE. These two algorithms worked even worse than the positioning based on a simple Geometrical restriction in our simulations.

### Summary

Non Line Of Sight (NLOS) condition strongly degrades the performance of subscriber positioning in wireless communication systems. Robust traditional algorithms, originally developed to use Line Of Sight (LOS) signal, fail in current dispersive scenarios. Several mitigation techniques have been proposed. Some of them consider the weighting of the available measures in order to get the better of each one. Others use geometrical restrictions to improve accuracy, and another group includes some lateral information to properly evaluate the quality of the measure and hence incorporate this data in the positioning process.

The most relevant approaches for positioning and NLOS mitigation have been simulated within realistic environments along this document for small sized cells. A new mitigation algorithm that considers both the weighting of measures and soft geometrical restrictions has been proposed for positioning based on TOA. This new algorithm makes a better work than those provided by the literature and used as reference. In fact, this new algorithm achieves a positioning error below 162 [m] for the 70% of the cases and below 191[m] for the 95% of the cases within a moderate dispersive environment ( $T_1=0.4$  [us]). These values are at least 10% lower than those provided by the reference WLLS algorithm. Therefore average positioning error decreases between 15 [m] and 45 [m] for this new method, around 35-70 [m] in comparison with Yi-Long approach, and around 50-70 [m] when it is compared with GALE. However, 163[m] is still high when it is compared with the 100 [m] required by the E911 regulation, and therefore new methods that better exploit the system characteristics to take

advantage of its dynamics and signal diversity will be the object of further research.

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**Annex. Derivation of the first two moments for the RMS Delay Spread defined as the Greenstein Model**

The derivation of these moments is eased by the fact that  $\xi$  in (14) is lognormal and therefore, it is related to  $\Xi$  as it is shown in (19), and admits to be expressed in relation to a new scaled variable  $\mathbf{z}$  with mean  $m_z$  and standard deviation  $\sigma_z$  as it is shown in (20):

$$\begin{aligned} \xi &= e^{\Xi \cdot \ln 10/10} = e^z \\ \text{with } \mathbf{z} &= \frac{\ln 10}{10} \Xi \end{aligned} \quad (19)$$

$$\begin{aligned} m_z &= E\{\mathbf{z}\} = \frac{\ln 10}{10} m_\xi \\ \sigma_z &= \sqrt{\text{var}\{\mathbf{z}\}} = \frac{\ln 10}{10} \sigma_\xi \end{aligned} \quad (20)$$

The computation of  $\xi$  may be related to the characteristic function  $\Phi_z(s)=E\{e^{zs}\}$  as follows:

$$\begin{aligned} E\{\xi\} &= E\{e^z\} = \Phi_z(s=1) \\ &= e^{s \cdot m_z + s^2 \cdot \sigma_z^2/2} \Big|_{s=1} = e^{m_z + \sigma_z^2/2} \end{aligned} \quad (21)$$

Similarly, the second moment may be performed as it is shown in (22):

$$\begin{aligned} E\{\xi^2\} &= E\{e^{2z}\} = \Phi_z(s=2) \\ &= e^{s \cdot m_z + s^2 \cdot \sigma_z^2/2} \Big|_{s=2} = e^{2m_z + 2\sigma_z^2} \end{aligned} \quad (22)$$

And the variance may be achieved as in (23):

$$\begin{aligned} \text{var}\{\xi\} &= E\{\xi^2\} - E^2\{\xi\} \\ &= e^{2m_z + \sigma_z^2} \left( e^{\sigma_z^2} - 1 \right) \end{aligned} \quad (23)$$

Furthermore, the statistics for the RMS delay spread are finally derived within (24) and (25):

$$E\{\tau_{rms}\} = T_1 d^\epsilon e^{m_z + \sigma_z^2/2} \quad (24)$$

$$\sigma_{\tau_{rms}} = E\{\tau_{rms}\} \sqrt{e^{\sigma_z^2} - 1} = k T_1 d^\epsilon \quad (25)$$