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The Graviton-Electron Vertex and The f^0 Meson Decay Width

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Abstract

It takes three form factors to describe the graviton-electron interaction. In the limit of zero momentum transfer these form factors essentially reduce to the mass of the electron. In this article we show that under certain conditions the form factors of the graviton-electron vertex function vanish at high energies. We first develop the exact massless spin-2^+ propagators in terms of two spectral functions. Working in a covariant gauge, which requires the use of an indefinite metric, we show that the spectral functions are positive definite. We then extend a theorem of Lehmann, Symansik, and Zimmermann, namely the vanishing of the π-N vertex function at high energies, to the graviton-electron interaction.

Keywords. graviton-electron interaction, quantum electrodynamics, Lehmann, Symansik, Zimmermann.

Vértice Gravitón-Electrón y Decaimiento del Mesón f^0

Resumen

Se necesitan tres factores de forma para describir la interacción gravitón-electrón. En el límite de transferencia de momento cero, estos factores de forma reducen esencialmente a la masa del electrón. En este artículo se muestra que bajo ciertas condiciones, los factores de forma de la función vértice gravitón-electrón se desvanecen a altas energías. Primero desarrollamos los propagadores spin-2^+ sin masa exacta en términos de dos funciones espectrales. Trabajando en un medidor de covariante, que requiere el uso de una métrica indefinida, se muestra que las funciones espectrales son definidas positivas. A continuación, extendemos un teorema de Lehmann, Symansik y Zimmermann, para conocer la desaparición de la función vértice π-N a altas energías a la interacción gravitón-electrón.

Palabras Clave. interacción gravitón-electrón, electrodinámica cuántica, Lehmann, Symansik, Zimmermann.

Introduction

The success of Quantum Electrodynamics (QED) as a massless spin-l field theory has provided a framework for studying theories of other spins. Recently, there has been considerable interest in investigating a massless spin-2^+ theory, particularly because of its connection with Einstein's gravitation theory. If such a theory is to describe gravitation it must predict a force that is long range, obeys the inverse square law, is attractive, and couples with all matter with equal strength.

R. P. Feynman [1] has shown that an interaction described by an exchange of massless spin-2^+ meson (graviton) yields a force in agreement with the above requirements. The theory, however, is considerable more complicated than QED because of the universality of the gravitational coupling. Stated differently, this says that whereas the photon is electromagnetically neutral, the graviton is not gravitationally neutral. The field equations describing gravity become highly non-linear having as a source the conserved energy-momentum tensor of matter and gravitation. B. Holstein has further studied the interaction of gravitons with matter [2] as well as G. Degrassi, et.al on fermion-graviton vertices [3].

In this study we study the interaction between gravitons and electrons, in particular, the asymptotic behavior of the form-factors describing the interaction. This interaction can be studied by looking at the matrix element $< p'|\theta_{\mu\nu}|p >$ where p' and p are momenta of the incoming and outgoing electron respectively and $\theta_{\mu\nu}$ is the conserved energy momentum tensor. This is as in Quantum Electrodynamics where the matrix element of interest is $< p'|j_\mu|p >$ and j_μ is a conserved current. Tha matrix element $< p'|\theta_{\mu\nu}|p >$ can be written as

$$\begin{aligned}\langle p'|\theta_{\mu\nu}|p\rangle = \bar{u}\left(p'\right)\big[&A_1\left(q^2\right)\delta_{\mu\nu} + A_2\left(q^2\right)p_\mu p_\nu \\ &+A_3\left(q^2\right)p'_\mu p'_\nu + A_4\left(q^2\right)p_{(\mu}p'_{\nu)} + A_5\left(q^2\right)p_{(\mu}\gamma_{\nu)} \\ &+A_6\left(q^2\right)p'_{(\mu}\gamma_{\nu)} + A_7\left(q^2\right)\left(\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu\right)\big]\,u\left(p\right),\end{aligned}$$

the $A_i\left(q^2\right)$ are invariant form factors and $q = p'-p$. We

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may drop the last term because of the anticommuting property of the γ matrices, i.e. $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. Hence this term is of the same type as the $A_1\left(q^2\right)$ term. Now we impose the conservation condition $\partial_\mu \theta_{\mu\nu}(x) = 0$, this gives

$$(p' - p)_\mu \langle p'|\theta_{\mu\nu}|p\rangle = 0,$$

Using the Dirac equation we arrive at the following condition

$$A_2\left(q^2\right) = A_3\left(q^2\right)$$

$$A_5\left(q^2\right) = A_6\left(q^2\right)$$

$$A_1\left(q^2\right) = \left(A_3\left(q^2\right) - A_4\left(q^2\right)\right)\left(p' \cdot p + m^2\right)$$

Substituting this in the first equation and redefining the form factors as follows

$$\frac{A_4(q^2)A_3(q^2)}{2} = \frac{G_3(q^2)}{m}$$
$$\frac{A_4(q^2)A_3(q^2)}{2} = \frac{G_2(q^2)}{m}$$
$$A_5\left(q^2\right) = G_1\left(q^2\right)$$

we obtain for the first equation

$$\langle p'|\theta_{\mu\nu}|p\rangle = \bar{u}\left(p'\right)\left[G_1\left(q^2\right)\left(\ell_\mu\gamma_\nu + \ell_\nu\gamma_\mu\right) + \frac{G_2(q^2)}{m}\ell_\mu\ell_\nu + \frac{G_3(q^2)}{m}\left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\right]u\left(p\right), \tag{1}$$

where the $G_i\left(q^2\right)$ area scalar form factors, m is the electron mass, $q = p' - p$ and $l = p' + p$. If we consider only the trace $\theta_{\mu\mu}$ we have

$$< p'|\theta_{\mu\nu}|p > = \bar{u}\left(p'\right) G\left(q^2\right) u\left(p\right), \tag{2}$$

where the $G\left(q^2\right)$ is a linear combination of the $G_i\left(q^2\right)$.

In the rest frame of the system and at $q^2 = 0$ the only surviving term in $\theta_{\alpha\alpha}$ is θ_{00} which is just the mass density of the electron. So we have

$$m = G(0), \tag{3}$$

where we have used the normalization condition given by $\frac{E}{m}\bar{u}\left(p\right)u\left(p'\right) = \delta_{pp'}$. Thus, if it is possible to calculate $G\left(q^2\right)$ then one can calculate the mass of the electron. Admittedly this is quite an ambitious task and in no way is it the aim of this thesis. However, the possibility of such a calculation with the use of unsubstracted dispersion relations certainly justifies the studying of the analytic properties of these form factors and in particular their asymptotic their asymptotic behavior.

H. Pagels [4] investigated the problem using the perturbation theory results for the asymptotic behavior of the vertex function, which showed a number of limitations [5]. It is, therefore, of interest to study the asymptotic behavior of these using only exact methods.

We shall approach the problem by extending a theorem of Lehmann, Symanzik, and Zimmerman [6] (LSZ) namely the vanishing of the π-nucleon vertex function at high energies, to the graviton-electron interaction. The case of the vertex function in Quantum Electrodynamics has been investigated by Evans [7].

It is necessary for the development of such theorem to have available the Lehmann-Källén [8] spectral representation of the propagators. These are well known in both spin-1 and in spin-0 theories. For this purpose we present a general formulation of a massless spin-2$^+$ theory in a covariant gauge and develop the exact propagators in terms of spectral functions. We work in a covariant gauge and an indefinite metric is necessary in order to insure that quantities such as the energy depend only on the two physically admissible graviton states. Quantizing [9] we are then able to prove that the spectral functions are positive definite. In order to avoid mathematical inconsistencies which appear from the start, we regularize using the method of regularization of Pauli and Villars [10]. With these preliminaries we prove that the graviton-electron vertex vanishes in the high energy limit.

In particular we consider the f^0 meson and calculate both the width of the f^0 decaying into two photons $\Gamma\left(f \to \gamma\gamma\right)$ as well as the photoproduction cross section for the f^0 meson. This is done by considering the process $N + \gamma \to f + N$ which consists of four types of interactions (Figure 1). By looking in certain kinematical regions and at small momentum transfers [11], we may neglect all but Figure 1c. Provided that the coupling constant remains approximately the same when the photon exchanged is on the mass shell, we can express the photoproduction cross section in terms of the width $\Gamma\left(f \to \gamma\gamma\right)$.

The spin-2$^+$ field

Field Equations

The Lagrangian for a free massless spin-2$^+$ field is

$$L = -\frac{1}{2}\frac{\partial g_{\mu\nu}\left(x\right)}{\partial x_\alpha}\frac{\partial g_{\mu\nu}\left(x\right)}{\partial x_\alpha}, \tag{4}$$

where $g_{\mu\nu}(x)$ is a symmetric field variable. By varying the above Lagrangian we obtain for the free field equation

$$\Box^2 g_{\mu\nu}(x) = 0 \tag{5}$$

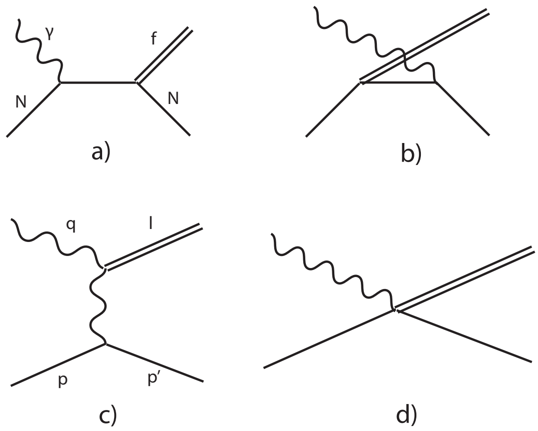

Figure 1: Diagrams for the process $\gamma + N \to f + N$.

We require that the field equations (5) be invariant under a gauge transformation of the type

$$g_{\mu\nu}\left(x\right) \to g^{'}_{\mu\nu}\left(x\right) = g_{\mu\nu}\left(x\right) + \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu \qquad (6)$$

where Λ_μ is an arbitrary vector. Since the fields $g_{\mu\nu}(x)$ are not uniquely determined we may impose a subsidiary condition equivalent to the Lorentz gauge in Electrodynamics, namely

$$\frac{\partial}{\partial x_\mu} g_{\mu\nu}\left(x\right) = 0 \qquad (7)$$

This restricts the gauge functions to those that satisfy,

$$\Box^2 \Lambda_\nu(x) = 0, \qquad (8)$$

$$\frac{\partial}{\partial x_\mu} \Lambda_\mu = 0 \qquad (9)$$

Indeed, with these conditions the field equations (5) are invariant under the gauge transformations in equation (6).

The tensor $g_{\mu\nu}(x)$ has sixteen independent components. By requiring that it can be symmetric we reduce the number of independent components to ten. The subsidiary condition (7) further reduces $g_{\mu\nu}(x)$ to six independent components. Finally, the gauge transformation (6) allows us only two independent degrees of freedom due to the four components of the arbitrary gauge function Λ_μ.

Quantization Rules

The Fourier decomposition of the field $g_{\mu\nu}(x)$ is given by

$$g_{\mu\nu}\left(x\right) = \frac{1}{\sqrt{2\omega V}} \sum_{\vec{k}} \left[a_{\mu\nu}\left(\vec{k}\right) e^{ikx} + a^{+}_{\mu\nu}\left(\vec{k}\right) e^{-ikx} \right] \qquad (10)$$

where a^{+} and a are creation and destruction operations respectively, ω is the energy, and V is the volume of periodicity. Following Gupta [9] we have for the quantization condition at arbitrary times,

$$\begin{aligned} \left[g_{\mu\nu}\left(x\right), g_{\alpha\beta}\left(y\right)\right] = -i \left(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \right. \\ \left. \delta_{\mu\nu}\delta_{\alpha\beta}\right) D\left(x-y\right) \end{aligned} \qquad (11)$$

where

$$D\left(x-y\right) = \frac{-i}{\left(2\pi\right)^3} \int dp e^{ip(x-y)} \epsilon\left(p\right) \delta\left(p^2\right)$$

and

$$\epsilon\left(p\right) = \frac{|p_0|}{p_0}, \quad dp = dp_1 dp_2 dp_3 dp_0$$

Furthermore, at equal times we have

$$\left[g_{\mu\nu}\left(x\right), g_{\alpha\beta}\left(y\right)\right]_{x_0=y_0} = 0 \qquad (12)$$

and

$$\begin{aligned} \left[g_{\mu\nu}\left(x\right), \dot{g}_{\alpha\beta}\left(y\right)\right]_{x_0=y_0} = -i \left(\delta_{\mu\alpha}\delta_{\beta\nu} + \delta_{\mu\beta}\delta_{\alpha\nu} - \right. \\ \left. \delta_{\mu\nu}\delta_{\alpha\beta}\right) \delta\left(\vec{x}-\vec{y}\right) \end{aligned} \qquad (13)$$

In this covariant quantization we have included ten spin-2^{+} states; however, there are only two physically meaningful states. We circumvent this difficulty by introducing an indefinite metric as in Quantum Electrodynamics in the Gupta-Bleuler gauge [12]. To this end the ten independent states are:

$$\begin{aligned} & a_{\pm}\left(\vec{k}\right) = \tfrac{1}{\sqrt{8}}\left[a_{11}\left(\vec{k}\right) - a_{22}\left(\vec{k}\right) \mp \tfrac{i}{\sqrt{2}} a_{12}\left(\vec{k}\right)\right] \\ & a^{'}_{30}\left(\vec{k}\right) = \tfrac{1}{2}\left[a_{33}\left(\vec{k}\right) + a_{00}\left(\vec{k}\right)\right] \\ & a\left(\vec{k}\right) = \tfrac{1}{\sqrt{8}}\left[a_{11}\left(\vec{k}\right) + a_{22}\left(\vec{k}\right) + a_{33}\left(\vec{k}\right) - a_{00}\left(\vec{k}\right)\right] \\ & a^{'}\left(\vec{k}\right) = \tfrac{1}{\sqrt{8}}\left[a_{11}\left(\vec{k}\right) + a_{22}\left(\vec{k}\right) + a_{33}\left(\vec{k}\right) + a_{00}\left(\vec{k}\right)\right] \\ & a_{31}\left(\vec{k}\right), \ a_{32}\left(\vec{k}\right), \ a_{02}\left(\vec{k}\right) \ \text{and} a_{10}\left(\vec{k}\right) \end{aligned} \qquad (14)$$

The indefinite metric operator is such that

$$\begin{aligned} & \left[\eta, a_{ij}\right] = \left[\eta, a'\right] = \left[\eta, a_{\pm}\right] = 0 \\ & \left\{\eta, a_{i0}\right\} = \left\{\eta, a\right\} = 0 \end{aligned} \qquad (15)$$

with i, j running from one to three. With this we have

$$\begin{aligned}
&\left[a_+\left(\vec{k}\right),a_+^+\left(\vec{k}\right)\right]=1 \quad &&\left[a_-\left(\vec{k}\right),a_-^+\left(\vec{k}\right)\right]=1\\
&\left[a_{13}\left(\vec{k}\right),a_{13}^+\left(\vec{k}\right)\right]=1 \quad &&\left[a_{10}\left(\vec{k}\right),a_{10}^+\left(\vec{k}\right)\right]=-1\\
&\left[a_{32}\left(\vec{k}\right),a_{32}^+\left(\vec{k}\right)\right]=1 \quad &&\left[a_{20}\left(\vec{k}\right),a_{20}^+\left(\vec{k}\right)\right]=-1\\
&\left[a_{30}^{,}\left(\vec{k}\right),a_{30}^{,+}\left(\vec{k}\right)\right]=1 \quad &&\left[a_{30}\left(\vec{k}\right),a_{30}^+\left(\vec{k}\right)\right]=-1\\
&\left[a'\left(\vec{k}\right),a'^+\left(\vec{k}\right)\right]=1 \quad &&\left[a\left(\vec{k}\right),a^+\left(\vec{k}\right)\right]=-1
\end{aligned} \tag{16}$$

The states of negative norm in this formalism can now be eliminated by choosing the subsidiary conditions

$$\frac{\partial}{\partial x_\mu}g_{\mu\nu}^+\left(x\right)\left|\psi\right\rangle=0 \tag{17}$$

where the + denotes positive frequency part and $|\psi\rangle$ is any state vector. By substituting equation (10) into equation (17) we have

$$\begin{aligned}
\left[a_{13}\left(\vec{k}\right)-a_{10}\left(\vec{k}\right)\right]\left|\psi\right\rangle=\left[a_{23}\left(\vec{k}\right)-a_{20}\left(\vec{k}\right)\right]\left|\psi\right\rangle=0\\
\left[a_{30}^{,}\left(\vec{k}\right)-a_{30}\left(\vec{k}\right)\right]\left|\psi\right\rangle=\left[a'\left(\vec{k}\right)-a\left(\vec{k}\right)\right]\left|\psi\right\rangle=0
\end{aligned} \tag{18}$$

Consider for example the Hamiltonian, given by

$$H=\sum_{\vec{k}}\omega\cdot\sum_{\lambda=1}^{10}N^{(\lambda)}\left(k\right) \tag{19}$$

where N is the number operator $N=a^+a$ and λ specifies the polarization state, this yields

$$\begin{aligned}
H=\sum_{\vec{k}}\omega\Big[&a_+^+\left(\vec{k}\right)a_+\left(\vec{k}\right)+a_-^+\left(\vec{k}\right)a_-\left(\vec{k}\right)\\
&+a_{13}^+\left(\vec{k}\right)a_{13}\left(\vec{k}\right)-a_{10}^+\left(\vec{k}\right)a_{10}\left(\vec{k}\right)\\
&+a_{23}^+\left(\vec{k}\right)a_{23}\left(\vec{k}\right)-a_{20}^+\left(\vec{k}\right)a_{20}\left(\vec{k}\right)\\
&+a_{30}^{,+}\left(\vec{k}\right)a_{30}^{,}\left(\vec{k}\right)-a_{30}^+\left(\vec{k}\right)a_{30}\left(\vec{k}\right)\\
&+a^{,+}\left(\vec{k}\right)a^{,}\left(\vec{k}\right)-a^+\left(\vec{k}\right)a\left(\vec{k}\right)\Big]
\end{aligned} \tag{20}$$

When applied to a state vector, by virtue of equation (18) we see that only the $a_{/pm}\left(\vec{k}\right)$ states contribute to the energy. Similarly, it can be shown that other observable quantities depend on only the physical gravitons. However, it must be noted, that when summing over intermediate states one must include all ten gravitons.

We introduce the interaction in the field equations through a conserved source $\theta_{\mu\nu}$ satisfying $\partial_\mu\theta_{\mu\nu}=0$. This tensor shall later be interpreted as the complete energy momentum tensor of matter and gravitation. The field equations for the interacting fields are then [13]

$$\Box^2 g_{\mu\nu}=-\theta_{\mu\nu}(x) \tag{21}$$

In what follows we shall refer to the free fields by a superscript (0) i.e. $\Box^2 g_{\mu\nu}^{(0)}(x)=0$ so as to distinguish them from the interacting fields introduced above. We have only specified that the tensor $\theta_{\mu\nu}$ be symmetric and conserved. Equation (21), however, may be non linear and $\theta_{\mu\nu}$ may depend on $g_{\mu\nu}$ itself.

Before proceeding we present a non-rigorous connection with general relativity.

Connection with Gravitation Theory

Until now we have only presented a mathematical framework for a spin-2^+ theory. We shall now attempt to establish a connection, in the classical limit, with Einstein's gravitation theory. For the purpose of this section only, we shall use the notation of Weinberg [12]. Einstein's equations are given by

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R_\lambda^\lambda=-8\pi G T_{\mu\nu} \tag{22}$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ is the stress tensor of matter without gravitation. G is the gravitational constant. In order to write these equations in flat space it is convenient to choose a coordinate system as Weinberg [14] such that the metric $g_{\mu\nu}$ approaches $\eta_{\mu\nu}(-1,1,1,1)$ the Minkowsky metric at large distances, that is

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \tag{23}$$

where $h_{\mu\nu}$ vanishes at infinity. However, $h_{\mu\nu}$ is not assumed to be small. Einstein's equations may now be written as

$$R_{\mu\nu}^{(1)}-\frac{1}{2}g_{\mu\nu}R_\lambda^{(1)\lambda}=-8\pi G\left[T_{\mu\nu}+t_{\mu\nu}\right] \tag{24}$$

where

$$R_{\mu\nu}^{(1)}=\tfrac{1}{2}\left(\Box^2 h_{\mu\nu}-\tfrac{\partial^2}{\partial x^\lambda\partial x^\mu}h_\nu^\lambda-\tfrac{\partial^2}{\partial x^\lambda\partial x^\nu}h_\mu^\lambda+\tfrac{\partial^2}{\partial x^\mu\partial x^\nu}h_\lambda^\lambda\right) \tag{25}$$

and

$$t_{\mu\nu}=\frac{1}{8\pi G}\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R_\lambda^\lambda-\left(R_{\mu\nu}^{(1)}-\frac{1}{2}\eta_{\mu\nu}R_\lambda^{(1)\lambda}\right)\right] \tag{26}$$

It should be emphasized that these are Einstein's equations and no approximations have been made.

We now may interpret $T_{\mu\nu} + t_{\mu\nu}$ as the total energy momentum tensor of matter and gravitation. Indeed, by virtue of the Bianchi identities

$$\frac{\partial}{\partial x^{\nu}}\left[R^{(1)\mu\nu} - \frac{1}{2}\eta^{\mu\nu}R^{(1)\lambda}_{\lambda}\right] = 0, \tag{27}$$

therefore,

$$\frac{\partial}{\partial x^{\nu}}\left(T^{\mu\nu} + t^{\mu\nu}\right) = 0 \tag{28}$$

If we define $\theta^{\mu\nu} = 8\pi G\left(T^{\mu\nu} + t^{\mu\nu}\right)$ we have

$$\partial_{\mu}\theta^{\mu\nu} = 0 \tag{29}$$

Furthermore, by defining

$$g_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\lambda}_{\lambda}, \tag{30}$$

and choosing the harmonic coordinate system

$$\frac{\partial}{\partial x^{\mu}}g^{\mu}_{\nu} = 0, \tag{31}$$

we have

$$\Box^2 g_{\mu\nu} = -\theta_{\mu\nu} \tag{32}$$

This is the same as equation (21). It should be noted, that in view of this interpretation, $\theta_{\mu\nu}$ is highly non linear. We thus identify the above $\theta_{\mu\nu}$ with the one in equation (21) and interpret it as the energy momentum tensor of matter and gravitation. In the above we have only established a point of contact between the two theories. Feynman [1], however, has shown that by starting with a spin-2^+ theory and invoking gauge invariance as well as energy momentum conservation, one arrives at Einstein's equations. Consider equation (32) with $T_{\mu\nu}$, the energy momentum tensor of matter only, as the source. The interaction between two matter fields with stress tensors $T_{\mu\nu}$ and $T'_{\alpha\beta}$ is given by

$$T^{,}_{\mu\nu}\frac{P_{\mu\nu\alpha\beta}}{k^2}T^{,}_{\alpha\beta}, \tag{33}$$

where

$$P_{\mu\nu\alpha\beta} = \delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \delta_{\mu\nu}\delta_{\alpha\beta}$$

This alone is sufficient to predict all the Newtonian effects of gravity in the non-relativistic limit. In addition it predicts a bending of light by the sun which is twice the Newtonian prediction and in agreement with Einstein. If one calculates, for example, the amplitude for Compton scattering of a graviton from a spinless particle, one finds by direct substitution of the gauge transformation given in equation (6) that the amplitude is not gauge invariant. The difficulty arises because the stress tensor $T_{\mu\nu}$ does not include the energy of the gravitational field, thus one has to add non-linear corrections to make the amplitude gauge invariant. At this point, however, $T_{\mu\nu}$ alone is no longer conserved. To correct this, one might define another tensor $T^{(1)}_{\mu\nu} = T_{\mu\nu} + \phi^{(0)}_{\mu\nu}$ so that $\partial_{\mu}T^{(1)}_{\mu\nu} = 0$. Repeating the process *n* times, each time invoking gauge invariance, one obtains $T^{(n)}_{\mu\nu} = T_{\mu\nu} + \phi^{n-1}_{\mu\nu}$ where $\partial_{\mu}T^{(n)}_{\mu\nu} = 0$. One would expect to arrive at the correct theory when *n* approaches infinity. From the Lagrangian point of view, this would imply an infinite number of terms. In fact, Feynman [1] shows that this series can be summed, when written in the mathematical language of general relativity, to yield Einstein's theory. Gauge invariance then becomes an invariance under general coordinate transformations.

Furthermore, we may interpret the tensor $\theta_{\mu\nu}$ in equation (32) as $T^{(\infty)}_{\mu\nu}$. With the above discussion in mind we base the rest of this work on the field equation (32). These equations are then highly non-linear, are gauge invariant and the source is conserved. We shall now call a massless spin-2^+ particle a graviton.

The matrix elements of $\theta_{\mu\nu}$ are not gauge invariant unless $\theta_{\mu\nu}$ depends on $g_{\alpha\beta}$ and $\dot{g}_{\alpha\beta}$ i.e., the theory must be non-linear. If, $\theta_{\mu\nu}$ is independent of $g_{\alpha\beta}$ and $\dot{g}_{\alpha\beta}$, as in the weak field approximation, we must then conclude, as is known, that the theory is not gauge invariant in this case [1].

The Massive Spin-2^+ Field

We conclude this study by presenting a brief formulation of massive spin-2^+ fields [15, 16]. The free Lagrangian density is given by

$$L = -\frac{1}{2}\frac{\partial f_{\mu\nu}(x)}{\partial x_{\alpha}}\frac{\partial f_{\mu\nu}(x)}{\partial x_{\alpha}} + \frac{1}{2}M^2 f_{\mu\nu}(x)\, f_{\mu\nu}(x), \tag{34}$$

where $f_{\mu\nu}(x)$ is the field variable and *M* is the mass of this field. The field equations are then

$$\left(\Box^2 - M^2\right) f_{\mu\nu}(x) = 0 \tag{35}$$

Because there are five degrees of freedom we also have

$$f_{\mu\nu}(x) - f_{\nu\mu}(x), \tag{36}$$

$$f_{\nu\nu}(x) = 0, \tag{37}$$

$$\partial_\mu f_{\mu\nu}(x) = 0 \tag{38}$$

The commutation relations are

$$\left[f_{\mu\nu}(x), f_{\alpha\beta}(y)\right] = \left(\xi_{\mu\alpha}\xi_{\nu\beta} + \xi_{\mu\beta}\xi_{\nu\alpha} - \frac{2}{3}\xi_{\mu\nu}\xi_{\alpha\beta}\right)\Delta(x-y), \tag{39}$$

where

$$\xi_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \tag{40}$$

and

$$\Delta(x) = -\frac{i}{(2\pi)^3}\int e^{ikx}\epsilon(k)\,\delta\left(k^2+M^2\right)dk$$

It should be noted that the term that multiplies $\Delta(x-y)$ in equation (39), which is essentially the sum over the polarizations, does not go over to the one appearing in equation (11) for the massless case as $M \to 0$. The fact that the massive spin-2^+ theory does not go over into a massless theory has been subject to considerable investigation [17–19]. We only note this difference and shall not consider the point further.

Finally, we introduce the interaction by a direct coupling of the field variable $f_{\mu\nu}(x)$ with the energy momentum tensor of the field in question $T_{\mu\nu}$ [20].

$$L_{\text{int}} = \frac{g}{M}T_{\mu\nu}f_{\mu\nu} \tag{41}$$

where *g* is a dimensionless coupling constant and *M* is the mass of the spin-2^+ field. The constant *M* has been introduced above only for dimensional reasons.

The resulting equation represents an adequate mathematical description of the graviton-electron interaction, which is requirement for the calculation of some processes involving spin-2^+ particles. In particular, the result shown in euation (41) has been employed in the very exact description of the the differential and total cross sections for the photoproduction of the f meson in terms of the width of the f0 decaying into two photons [21].

Conclusions

In the present article, a mathematical formulation for describing the graviton-electron interaction without recurring to significant mathematical approximations is presented. In particular, we first develop the exact massless spin-2^+ propagators in terms of two spectral functions. Then, working in a covariant gauge which requires the use of an indefinite metric, we show that the spectral functions are positive definite. Finally, we extend a theorem of Lehmann, Symansik, and Zimmermann, namely the vanishing of the π-N vertex function at high energies, to the graviton-electron interaction. The theoretical development introduced here is proposed as a general approach for describing processes involving spin-2^+ particles.

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